647510 **MSC2MatC204x** **Seat No :_____**

M.Sc.(Maths.) Semester - 2 (*CBCS*) Examination
April/May-2023 (NEW COURSE)
Methods in Partial Differential Equations (Core)

Time: 2:30 Hours **Marks: 70**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Q.1(A) Answer the following questions. (Any Two) **(10)**

1. A necessary and sufficient condition that there exists between two functions $u(x,y)$ and $v(x,y)$, a relation $F(u,v)=0$ not involving x or y explicitly is that $\frac{\partial(u,v)}{\partial(x,y)}=0$.

2. Let $X=(P,Q,R)$ where P,Q,R are functions of x,y,z.

If $X\cdot curlX=0$ then the pfaffian differential equation $Pdx+Qdy+Rdz=0$ is integrable.

3. Find the general solution of the partial differential equation $z(xp-yq)=y^2-x^2$.

Q.1(B) Answer the following question. (Any One) **(04)**

1. Find the solution of homogeneous pfaffian differential equation $yz\,dx+xz\,dy+xy\,dz=0$.

2. Find the solution of the pfaffian differential equation $(y+z)dx+(z+x)dy+(x+y)dz=0$.

Q.2(A) Answer the following questions. (Any Two) **(10)**

1. Find the integral surface of the linear partial differential equation $x(y^2+z)p-y(x^2+z)q=(x^2-y^2)z$ which contains the straight line $x+y=0, z=1$.

2. Find the surface which intersects the surfaces of the system $z(x+y)=c(3z+1)$ orthogonally and which passes through the circle $x^2+y^2=1, z=1$.

3. Find the equation of the integral surface of the differential equation $2y(z-3)p+(2x-z)q=y(2x-3)$ which passes through the circle $z=0,\ x^2+y^2=2x$.

Q.2(B) Answer the following question. (Any One) **(04)**

1. Eliminate arbitrary function F from the following equation and find the corresponding partial differential equation $z=xy+F(x^2+y^2)$.

2. Find the surface which is orthogonal to the one parameter system $z=cxy(x^2+y^2)$ and which passes through the hyperbola $z=0,\ x^2-y^2=a^2$.

Q.3(A) Answer the following questions. (Any Two) **(10)**

1. Find the solution of the equation $z = \frac{1}{2}(p^2 + q^2) + (p - x)(q - y)$ which passes through x-axis.
2. The equations $f(x, y, z, p, q) = 0$ & $g(x, y, z, p, q) = 0$ are compatible then $[f, g] = \frac{\partial(f,g)}{\partial(x,p)} + p\frac{\partial(f,g)}{\partial(z,p)} + \frac{\partial(f,g)}{\partial(y,q)} + q\frac{\partial(f,g)}{\partial(z,q)} = 0$.
3. Show that the equations $xp = yq$ and $z(xp + yq) = 2xy$ are compatible and solve them.

Q.3(B) Answer the following question. (Any One) **(04)**

1. Solve the partial differential equation $p + q = pq$.
2. Solve the partial differential equation $z = px + qy + p^2 + q^2$.

Q.4(A) Answer the following questions. (Any Two) **(10)**

1. Let u be a C.F. and z_1 be a P.I. of equation $F(D, D')z = f(x, y)$ where $F(D, D') = \sum_{r=0}^{n}\sum_{s=0}^{m} C_{rs}\, D^r D'^s$ then the general solution of equation is $u + z_1$.
2. Let $(\alpha D + \beta D' + \gamma)$ be a factor of $F(D, D')$ with $\alpha \neq 0$. Then $u = e^{-\frac{\gamma x}{\alpha}}\, \emptyset(\beta x - \alpha y)$ is a solution of $F(D, D')z = 0$ where $\emptyset$ is an arbitrary function.
3. Solve the partial differential equation $(D^2 - D'^2)z = x - y$.

Q.4(B) Answer the following question. (Any One) **(04)**

1. Solve the partial differential equation $(2D + D' + 1)(D - D' - 1)z = 0$.
2. Solve the partial differential equation $(D^2 - DD' - 6D'^2)z = 0$.

Q.5(A) Answer the following questions. (Any Two) **(10)**

1. Verify that the following differential equation is integrable and find its primitive. $(y^2 + yz)dx + (xz + z^2)dy + (y^2 - xy)z = 0$.
2. Solve the following partial differential equation $(x^2D^2 - y^2D'^2 + xD - yD')z = 0$.
3. Solve the partial differential equation by using charpit's method $(p^2 + q^2)y = qz$.

Q.5(B) Answer the following question. (Any One) **(04)**

1. Eliminate arbitrary functions f and g from $z = f(x + ay) + g(x - ay)$ where a is constant and obtain partial differential equation.
2. Show that the equations $xp - yq = x$ and $x^2p + q = xz$ are compatible.

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